

College Algebra

Concise notes, formulas, rules, and plain-language reminders for each topic.

Module 1: Function Foundations Through Linear Functions

Students learn the basic language of functions through graphs, tables, equations, and linear models. Transformations are kept minimal here.

Module 1A Function Notation, Graphs, and Tables

Build the basic language for reading and describing functions from notation, tables, and graphs.

IN PLAIN TERMS

A function is a rule that matches each input to one output. This section is about reading the story a function tells from its notation, graph, and table.

KEY PARTS

- $f(3)$ means the output when the input is 3. It does not mean $f \times 3$.
- Domain is the set of allowed inputs. Range is the set of outputs the function actually makes.
- Zeros are x-values where $f(x) = 0$. Intercepts are where the graph hits an axis.

RULES AND FORMULAS

- Average rate of change from $x = a$ to $x = b$:
$$\frac{f(b) - f(a)}{b - a}$$
- Increasing means outputs rise as you move right. Decreasing means outputs fall as you move right.
- Interval notation reminder: $x > 2$ becomes $(2, \infty)$, while $x \leq 2$ becomes $(-\infty, 2]$.

LOOK FOR: whether each input has exactly one output and whether the graph or table gives the domain, range, intercepts, and interval behavior directly.

Module 1B Linear Equations and Inequalities

Solve linear relationships analytically and represent solution sets clearly.

IN PLAIN TERMS

An equation asks for the value that makes both sides match. An inequality asks where one side stays bigger or smaller than the other.

KEY PARTS

- Do the same operation to both sides to keep the balance true.
- Use a number line to show where an inequality is true, not just a single point.
- Context matters in applications because negative or fractional answers are not always reasonable.

RULES AND FORMULAS

- If you multiply or divide by a negative number, reverse the inequality sign.
- Interval notation: $x > 3$ is $(3, \infty)$, and $x \leq 3$ is $(-\infty, 3]$.
- Check a test value when you are not sure whether a region should be shaded or included.

LOOK FOR: the operation that isolates the variable most directly, and whether the answer should be one value, an interval, or no solution.

Module 1C Linear Functions and Graphs

Connect slopes, intercepts, equations, and graphs as one function story.

IN PLAIN TERMS

A linear function is a straight-line pattern. Its main story is how fast it changes and where it starts.

KEY PARTS

- Slope tells steepness and direction. Positive slope rises right. Negative slope falls right.
- The y-intercept is where the line crosses the y-axis. The x-intercept is where the output is 0.
- For a line, average rate of change stays constant everywhere.

RULES AND FORMULAS

- Slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Slope-intercept form: $y = mx + b$
- Point-slope form: $y - y_1 = m(x - x_1)$

LOOK FOR: a constant change pattern, the meaning of the slope units, and whether the intercepts make sense in the situation.

Module 1D Linear Modeling and Applications

Apply linear functions to prediction, rate, and model interpretation.

IN PLAIN TERMS

Modeling means turning a real situation into an equation that can predict, compare, or explain what is happening.

KEY PARTS

- The slope is the rate per 1 unit of input. The intercept is the starting amount.
- A break-even idea usually means the outputs of two models are equal at the same input.
- A good model also has reasonable units and a reasonable input interval.

RULES AND FORMULAS

- Basic linear model: $y = mx + b$
- Prediction works best inside the range of data that created the model.
- Always state what the variables stand for before solving.

LOOK FOR: what the slope means in words, what the intercept means at input 0, and whether your prediction is realistic.

Module 2: Quadratic Equations and Quadratic Functions

Students extend function analysis from lines to curves. This is where transformations become more formal, but only through quadratics.

Module 2A Factoring and Quadratic Equations

Solve quadratic equations first through structure and factoring.

IN PLAIN TERMS

Factoring rewrites a quadratic as multiplication, and that matters because products are easy to set equal to 0.

KEY PARTS

- Set the quadratic equal to 0 before using factoring to solve.
- Factor out any GCF first so the rest is easier to see.
- Zeros of the equation become x-intercepts of the graph when the quadratic is written as a function.

RULES AND FORMULAS

- Zero product property: if $uv = 0$, then $u = 0$ or $v = 0$.
- Square root property: if $x^2 = k$, then $x = \pm\sqrt{k}$.
- Difference of squares: $a^2 - b^2 = (a - b)(a + b)$

LOOK FOR: whether the equation is in standard form and whether the factors truly multiply back to the original quadratic.

Quadratic Formula, Completing the Square, and Complex Numbers

Expand solving tools and interpret the type of solutions a quadratic can produce.

IN PLAIN TERMS

When factoring is awkward or impossible, completing the square and the quadratic formula still let you solve the equation and tell what kind of answers to expect.

KEY PARTS

- Completing the square rewrites a quadratic around a perfect-square pattern.
- The discriminant tells whether the answers are two real numbers, one real number, or a complex pair.
- If a graph never crosses the x-axis, the real-number graph has no real zeros.

RULES AND FORMULAS

- Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Discriminant: $D = b^2 - 4ac$. If $D > 0$, two real solutions. If $D = 0$, one repeated real solution. If $D < 0$, two complex solutions.
- Complex number reminder: $i^2 = -1$

LOOK FOR: the sign of the discriminant and whether the problem wants exact answers, decimals, or a graph-based interpretation.

Module 2C Quadratic Functions and Graphs

Read quadratics through vertex form, graph features, and transformations.

IN PLAIN TERMS

A quadratic graph is a parabola. The main things to read are the turning point, the direction it opens, and where it crosses the axes.

KEY PARTS

- The vertex is the highest or lowest point, depending on whether the parabola opens down or up.
- The axis of symmetry is the vertical line through the vertex.
- Transformations shift, stretch, reflect, or shrink the parent parabola.

RULES AND FORMULAS

- Vertex form: $y = a(x - h)^2 + k$, so the vertex is (h, k) .
- Axis of symmetry: $x = h$ in vertex form, or $x = -\frac{b}{2a}$ in standard form.
- If $a > 0$, the graph opens up. If $a < 0$, it opens down.

LOOK FOR: the vertex first, then intercepts, then the intervals where the graph rises or falls.

Module 2D Quadratic Inequalities and Applications

Use intervals, graphs, and applications to reason about quadratic behavior.

IN PLAIN TERMS

Quadratic inequalities ask where the parabola is above, below, on, or between certain values. Applications often use the vertex as a best or worst value.

KEY PARTS

- Zeros split the number line into intervals where the quadratic keeps one sign.
- A graph shows quickly where the expression is positive or negative.
- In applications, the vertex often gives a maximum or minimum value.

RULES AND FORMULAS

- Use test points after finding the zeros to decide which intervals satisfy the inequality.
- Use parentheses for values not included and brackets only when the endpoint is included and defined.
- Optimization usually means find the vertex and explain what the input and output mean.

LOOK FOR: whether the problem wants where the quadratic is above 0, below 0, or at a highest or lowest value in context.

Module 3: Absolute Value and Radical Functions

Students add new function behaviors: sharp turns, restricted domains, and extraneous solutions. Transformations are reinforced through these specific function types.

Module 3A Absolute Value Equations, Inequalities, and Functions

Introduce sharp-turn graphs and analytic work with absolute value equations and inequalities.

IN PLAIN TERMS

Absolute value measures distance, so the graph makes a V-shape and equations often split into two cases that are the same distance away.

KEY PARTS

- $|x|$ is distance from 0, so it can never be negative.
- Absolute value graphs have a vertex and two linear sides.
- Piecewise thinking helps because the expression inside the bars changes sign.

RULES AND FORMULAS

- Equation rule: if $|x - a| = b$, then $x - a = b$ or $x - a = -b$.
- Graph form: $y = a|x - h| + k$, so the vertex is (h, k) .
- If $a < 0$, the V opens down. If $a > 0$, it opens up.

LOOK FOR: the vertex, the two-case structure, and whether the problem asks for values inside or outside a distance range.

Module 3B Radical Equations and Radical Functions

Add domain restrictions, extraneous solutions, and radical graph behavior.

IN PLAIN TERMS

Radicals undo powers. Square roots create domain limits, and solving radical equations often makes extra answers that need to be checked.

KEY PARTS

- Even roots need the inside to be nonnegative in the real-number system.
- Square both sides only after isolating the radical when possible.
- Cube root functions are more flexible because negative inputs are allowed.

RULES AND FORMULAS

- $x^{1/2} = \sqrt{x}$ and $x^{1/3} = \sqrt[3]{x}$
- For $y = \sqrt{x - h} + k$, the graph starts at (h, k) and the real-number domain is $x \geq h$.
- Always plug answers back into the original equation to catch extraneous solutions.

LOOK FOR: the domain restriction before solving and whether squaring both sides created an answer that does not really work.

Module 4: Polynomial Equations, Inequalities, and Functions

Students generalize from quadratics to higher-degree polynomials. The major new ideas are end behavior, multiplicity, polynomial division, complex zeros, and polynomial inequalities.

Module 4A Polynomial Foundations and Factoring

Prepare polynomial work by reading structure, degree, and factoring patterns.

IN PLAIN TERMS

Polynomials are built from whole-number powers of a variable. Before graphing or solving them, you need to recognize their structure and factoring patterns.

KEY PARTS

- Degree is the largest exponent. The leading coefficient is the coefficient of the highest-degree term.
- Factor a GCF first because it often exposes the next pattern quickly.
- Grouping and special products are common tools once a simple GCF is gone.

RULES AND FORMULAS

- Difference of squares: $a^2 - b^2 = (a - b)(a + b)$
- Perfect square trinomials: $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$
- Sum and difference of cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

LOOK FOR: the largest common factor first, then a familiar pattern such as squares, grouping, or cubes.

Module 4B Polynomial Division and Zeros

Use division and zero theorems to uncover factors and roots.

IN PLAIN TERMS

Division lets you strip away known factors so a hard polynomial becomes a simpler one. This is how you hunt for zeros when factoring is not obvious.

KEY PARTS

- Long division works with any divisor. Synthetic division is a shortcut when dividing by $x - c$.
- A zero of the polynomial creates a factor of the form $x - c$.
- Possible rational zeros come from constant factors over leading-coefficient factors.

RULES AND FORMULAS

- Remainder theorem: dividing $f(x)$ by $x - c$ leaves remainder $f(c)$.
- Factor theorem: if $f(c) = 0$, then $x - c$ is a factor.
- Rational zero theorem: possible rational zeros are $\pm \frac{p}{q}$, where p divides the constant term and q divides the leading coefficient.

LOOK FOR: easy rational candidates first, then use division to lower the degree and repeat.

Module 4C Polynomial Graphs, Complex Zeros, and Multiplicity

Tie zero structure to graph shape, end behavior, and multiplicity.

IN PLAIN TERMS

Polynomial graphs are shaped by their zeros and their end behavior. Multiplicity tells whether the graph crosses the axis or just touches it and turns.

KEY PARTS

- Odd multiplicity usually means the graph crosses the x-axis. Even multiplicity usually means it touches and turns.
- The degree and leading coefficient control the long-run left and right end behavior.
- Complex zeros of real-coefficient polynomials come in conjugate pairs.

RULES AND FORMULAS

- For odd degree with positive leading coefficient: left end down, right end up.
- For even degree with positive leading coefficient: both ends up. With negative leading coefficient: both ends down.
- Polynomial domain is all real numbers. Range often needs a graph or turning-point analysis.

LOOK FOR: the zeros first, then their multiplicities, then the left and right end behavior.

Module 4D Polynomial Inequalities

Solve inequality questions using sign intervals and critical values.

IN PLAIN TERMS

A polynomial inequality asks where the graph is above or below a target level. The answer is usually an interval or several intervals.

KEY PARTS

- Find the zeros first because they split the number line into sign regions.
- Polynomials are defined everywhere, so only zeros matter as interval split points.
- A graph can verify which intervals are above or below the axis.

RULES AND FORMULAS

- Use a test point from each interval or analyze multiplicity to decide the sign.
- Include zeros only when the inequality allows equality, such as \geq or \leq .
- Write the final answer in interval notation, not just as a shaded sketch.

LOOK FOR: which intervals make the expression positive or negative and whether endpoints belong in the answer.

Module 4E Polynomial Modeling

Interpret polynomial models and decide what the features mean in context.

IN PLAIN TERMS

A polynomial model is only useful if you can explain what its zeros, intercepts, and long-run behavior mean in the real situation.

KEY PARTS

- Zeros often mean break-even points, stopping points, or times when a quantity becomes 0.
- The y -intercept is the modeled output at input 0.
- Predictions far beyond the data range are risky because the model may stop matching reality.

RULES AND FORMULAS

- Interpret every number with units and context before calling it important.
- Use graph features and algebra together. One should support the other.
- If regression is used, compare the model to actual data before trusting a prediction.

LOOK FOR: whether the graph feature you found has a clear real-world meaning and whether the model is reasonable only on a certain interval.

Module 5: Rational Expressions, Equations, Inequalities, and Functions

Students move from polynomial behavior to ratios of polynomials. The major new ideas are domain restrictions, holes, vertical asymptotes, horizontal asymptotes, and rational inequalities.

Module 5A Rational Expressions and Domain Restrictions

Build rational-expression fluency while tracking restrictions from the start.

IN PLAIN TERMS

Rational expressions are fractions with polynomials. The main caution is that anything making the denominator 0 is banned from the start.

KEY PARTS

- Factor first before canceling anything.
- A factor that cancels out still creates a restricted value in the original expression.
- LCD means least common denominator and is the key for adding or subtracting fractions.

RULES AND FORMULAS

- Domain restriction rule: denominator $\neq 0$
- To divide fractions, multiply by the reciprocal of the second fraction.
- For adding or subtracting, rewrite each fraction over the LCD before combining numerators.

LOOK FOR: denominator restrictions before any simplification and whether a factor really cancels or only looks similar.

Module 5B Rational Equations

Solve rational equations carefully and check for invalid solutions.

IN PLAIN TERMS

Rational equations are usually solved by clearing denominators, but that shortcut can create answers that make an original denominator equal to 0.

KEY PARTS

- Write the restricted values first so you know what can never be an answer.
- Multiply both sides by the LCD to remove fractions efficiently.
- After solving, check each answer in the original equation, not just the simplified one.

RULES AND FORMULAS

- LCD method: multiply every term on both sides by the least common denominator.
- An answer is extraneous if it solves the transformed equation but not the original rational equation.
- Keep track of units in applications because rates and times often appear as rational expressions.

LOOK FOR: the restricted values first and a final substitution check before you box an answer.

Module 5C Rational Functions and Graphs

Read rational graphs through holes, asymptotes, intercepts, and behavior.

IN PLAIN TERMS

Rational function graphs can break, jump toward asymptotes, or have missing points. Their algebra tells you where those features happen.

KEY PARTS

- Zeros come from the numerator equaling 0 after canceling common factors.
- Vertical asymptotes come from uncanceled denominator zeros.
- A canceled factor creates a hole instead of a vertical asymptote.

RULES AND FORMULAS

- If degree numerator < degree denominator, horizontal asymptote is $y = 0$.
- If the degrees are equal, horizontal asymptote is the ratio of leading coefficients.
- If degree numerator > degree denominator, there is no horizontal asymptote, though another end-behavior model may exist.

LOOK FOR: restrictions, holes, and vertical asymptotes before sketching or interpreting the graph.

Module 5D Rational Inequalities

Use interval reasoning to solve rational inequalities.

IN PLAIN TERMS

Rational inequalities work like polynomial inequalities, but now both zeros and undefined values split the number line.

KEY PARTS

- Find zeros and denominator restrictions first because both create critical points.
- A sign chart is often cleaner than trying to solve the inequality in one big step.
- Graphs can confirm where the expression is above or below 0.

RULES AND FORMULAS

- Undefined values are never included in the answer, even with \leq or \geq .
- Zeros are included only when equality is allowed and the function is defined there.
- Write the final answer as one or more intervals after testing the sign on each region.

LOOK FOR: both kinds of critical values and whether the answer should include zeros, exclude holes, or both.

Module 6: Function Operations, Composition, and Inverses

Students now have enough function experience to combine functions and reverse functions in a meaningful way. This module prepares directly for logarithms and later inverse trig.

Module 6A Algebra of Functions

Combine functions carefully and keep domain restrictions in view.

IN PLAIN TERMS

Functions can be added, subtracted, multiplied, and divided like expressions, but the allowed inputs now depend on both functions at once.

KEY PARTS

- Operations on functions create a new function with its own formula.
- The new domain is the overlap of inputs that work in every part of the expression.
- Division is the most restrictive case because the divisor function cannot equal 0.

RULES AND FORMULAS

- $(f + g)(x) = f(x) + g(x)$
- $(f - g)(x) = f(x) - g(x)$, $(fg)(x) = f(x)g(x)$
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, with $g(x) \neq 0$

LOOK FOR: the overlap of domains and any value that makes a denominator 0 after combining the functions.

Module 6B Composition of Functions

Track layered inputs and outputs through composed functions.

IN PLAIN TERMS

A composition feeds the output of one function into another. Think of it as a two-step machine where order matters.

KEY PARTS

- In $f(g(x))$, do the inside function g first, then apply f .
- The output of the inside function must be allowed as an input for the outside function.
- Compositions often model multi-step processes in real situations.

RULES AND FORMULAS

- Composition notation: $(f \circ g)(x) = f(g(x))$
- To find the domain, keep only x -values where the inside works and its output also works in the outside.
- Switching the order usually changes the result: $f(g(x))$ and $g(f(x))$ are usually different.

LOOK FOR: the inside output first and then whether that output is legal for the outside function.

Module 6C Inverse Functions

Reverse function processes and connect inverse pairs by graph and formula.

IN PLAIN TERMS

An inverse function undoes the original function. You can only have a true inverse when each output comes from just one input.

KEY PARTS

- One-to-one means no repeated outputs for different inputs.
- The graph of an inverse is a reflection across the line $y = x$.
- Some functions need a restricted domain before they can have an inverse that is still a function.

RULES AND FORMULAS

- Horizontal line test: if any horizontal line hits the graph more than once, the function is not one-to-one.
- To find an inverse, write $y = f(x)$, swap x and y , then solve for y .
- Check with composition: $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

LOOK FOR: whether the original graph passes the horizontal line test and whether the inverse needs a domain restriction first.

Module 6D Difference Quotient and Rate of Change

Use the difference quotient as light preparation for later calculus work.

IN PLAIN TERMS

The difference quotient measures how much the output changes compared with a small input change. It is a bridge into future calculus ideas.

KEY PARTS

- Average rate of change compares two points on a function.
- The difference quotient uses a general step size h instead of specific endpoints.
- Simplifying the expression carefully is usually the hardest part.

RULES AND FORMULAS

- Difference quotient: $\frac{f(x+h) - f(x)}{h}$, with $h \neq 0$
- Average rate of change from a to b : $\frac{f(b) - f(a)}{b - a}$
- Cancel h only after it is a common factor in the numerator.

LOOK FOR: whether the numerator can factor or simplify cleanly before you divide by h .

Module 7: Exponential and Logarithmic Functions

Students study non-polynomial growth and inverse relationships. Exponent rules are reviewed just in time, and logarithms build naturally from inverse functions.

Module 7A Exponent Rules and Exponential Functions

Review exponents and build the graph story for growth and decay.

IN PLAIN TERMS

Exponential functions change by a constant factor, not a constant difference. That is why they grow or decay much faster than linear functions.

KEY PARTS

- Growth means the base is bigger than 1. Decay means the base is between 0 and 1.
- The y-intercept is the starting value when $x = 0$.
- The basic exponential graph approaches, but never hits, its horizontal asymptote.

RULES AND FORMULAS

- Exponent rules: $a^m a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$,
 $(a^m)^n = a^{mn}$
- Negative and rational exponents: $a^{-n} = \frac{1}{a^n}$,
 $a^{m/n} = \sqrt[n]{a^m}$
- Basic form: $y = ab^x$, where a is the starting value and b is the growth or decay factor.

LOOK FOR: whether the pattern changes by multiplying, not adding, and whether the base signals growth or decay.

Module 7B Logarithmic Functions

Treat logarithms as inverse functions with their own graph behavior.

IN PLAIN TERMS

A logarithm answers the question: what power do I put on the base to get this number? That is why logs are the inverse of exponentials.

KEY PARTS

- Logarithmic and exponential forms say the same thing in different languages.
- The input of a log must be positive in the real-number system.
- A logarithmic graph has a vertical asymptote instead of a horizontal one.

RULES AND FORMULAS

- Conversion rule: $\log_b(a) = c$ means $b^c = a$.
- Common log means base 10. Natural log means base e .
- For $y = \log_b(x - h) + k$, the vertical asymptote shifts to $x = h$.

LOOK FOR: a positive input inside the log and the inverse relationship with an exponential graph or equation.

Module 7C Properties of Logarithms

Use log rules as tools for rewriting and solving expressions.

IN PLAIN TERMS

Log rules turn multiplication into addition and powers into front coefficients. They are rewriting tools, not random shortcuts.

KEY PARTS

- Expanding means breaking one log into several smaller logs.
- Condensing means combining several logs into one.
- These rules work only for products, quotients, and powers, not for sums inside a log.

RULES AND FORMULAS

- Product rule: $\log_b(MN) = \log_b M + \log_b N$
- Quotient rule: $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$
- Power rule: $\log_b(M^p) = p \log_b M$, and change of base: $\log_b M = \frac{\ln M}{\ln b}$

LOOK FOR: products, quotients, and powers only. Do not try to split $\log(x + 3)$ into two logs.

Module 7D Exponential and Logarithmic Equations

Solve equations analytically and confirm results with graph or table support.

IN PLAIN TERMS

To solve exponential or logarithmic equations, rewrite both sides in a compatible form, then isolate the unknown. Log equations still need a domain check at the end.

KEY PARTS

- If both sides can be written with the same base, match the exponents.
- If not, use logarithms to bring the exponent down where you can solve it.
- For log equations, every final answer must keep the log input positive.

RULES AND FORMULAS

- Same-base rule: if $b^u = b^v$, then $u = v$.
- Log equation method: isolate the log, then exponentiate or rewrite in exponential form.
- Graphs and tables are useful for checking whether the solution makes sense numerically.

LOOK FOR: a common base, an isolated log, and a final domain check so no invalid answer slips through.

Module 7E Exponential and Logarithmic Modeling

Model growth, decay, finance, and related real-world behavior.

IN PLAIN TERMS

Exponential and logarithmic models describe repeated percent change, investment growth, decay, and situations where the rate depends on the current amount.

KEY PARTS

- The initial value is the amount at time 0. The growth or decay factor controls how fast the amount changes.
- Half-life and doubling time are just special times when the output reaches a target amount.
- Model limits matter because real data do not keep following the same pattern forever.

RULES AND FORMULAS

- Discrete model: $A = ab^t$
- Compound interest: $A = P \left(1 + \frac{r}{n}\right)^{nt}$
- Continuous growth or decay: $A = Pe^{rt}$

LOOK FOR: the starting value, the repeated growth or decay factor, and whether the problem is discrete, compounded, or continuous.

Module 8: Systems of Equations and Partial Fractions

Students solve systems analytically, graphically, and numerically, then apply systems to partial fraction decomposition.

Module 8A Systems of Equations Graphically and Numerically

Start system solving by interpreting intersections and verifying solutions.

IN PLAIN TERMS

A system asks where two equations are true at the same time. Graphically, that means where their graphs intersect.

KEY PARTS

- One intersection means one solution. No intersection means no solution. The same graph means infinitely many solutions.
- Tables help estimate where curves or lines meet when an exact method is awkward.
- A system solution is usually an ordered pair, not a single number.

RULES AND FORMULAS

- Consistent means at least one solution. Inconsistent means no solution.
- Independent means exactly one solution. Dependent means infinitely many solutions.
- Always test the solution in both equations if you solved numerically or by table.

LOOK FOR: the intersection meaning first, then decide whether a graph, table, or algebraic method is the most efficient way to find it.

Module 8B Systems by Algebraic Methods

Use substitution and elimination for direct algebraic solving.

IN PLAIN TERMS

Algebraic methods solve systems exactly. Substitution replaces one variable, while elimination removes one variable by adding or subtracting equations.

KEY PARTS

- Substitution is fast when one equation already solves neatly for a variable.
- Elimination is strong when coefficients line up or can be made to line up easily.
- Nonlinear systems often use substitution so one equation can be plugged into the other.

RULES AND FORMULAS

- After finding one variable, substitute back to get the matching partner value.
- When eliminating, multiply one or both equations first if needed so a variable cancels cleanly.
- Final answers must satisfy every equation in the system.

LOOK FOR: the method that creates the least algebra and whether the resulting ordered pair works in both equations.

Module 8C Systems Applications

Translate applied settings into systems and interpret the results.

IN PLAIN TERMS

Applications turn a word problem into two equations that describe the same situation from different angles. The meaning of the answer matters more than the algebra alone.

KEY PARTS

- Define variables clearly before writing any equation.
- Break-even means two models are equal at the same input.
- Mixture and value problems usually combine totals and part-by-part equations.

RULES AND FORMULAS

- Total value pattern: quantity \times rate or price = contribution to the total.
- Use the final solution sentence to explain what each coordinate or variable value means.
- Reject answers that break the real situation, such as negative counts or impossible percentages.

LOOK FOR: the variables, the total relationships, and whether the final answer is realistic in context.

Module 8D Partial Fraction Decomposition

Use systems work to support partial fraction decomposition.

IN PLAIN TERMS

Partial fractions rewrite one complicated rational expression as a sum of simpler fractions. The simpler pieces are easier to integrate later and easier to analyze now.

KEY PARTS

- Decompose only after the denominator is fully factored.
- If the numerator degree is too large, divide first so the fraction is proper.
- A system often appears when you compare coefficients to find the unknown constants.

RULES AND FORMULAS

- For distinct linear factors: $\frac{P(x)}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$
- For repeated linear factors, use a term for each power, such as $\frac{A}{x-a} + \frac{B}{(x-a)^2}$.
- For irreducible quadratic factors, use a linear numerator such as $\frac{Ax+B}{x^2+1}$.

LOOK FOR: whether the fraction is proper, whether the denominator is fully factored, and what kind of pieces the factors require.