Quantitative Reasoning

Course modules and practice resources in one place.

Module 1: Measurement

Measurement practice and worked examples on unit conversions and applied measurement problems.

VIDEO EXAMPLE

How many hectometers are there in 4357 cm?

SCAN FOR VIDEO WALKTHROUGH
youtu.be/F9z7aQTqCpA?si=Fk6lwckgYm9QFAms&t=220

Answer: 0.4357 hm

VIDEO EXAMPLE

Convert 33.01 kg to g.

SCAN FOR VIDEO WALKTHROUGH
youtu.be/F9z7aQTqCpA?si=7gseRl_TQH_4l7Dl&t=176

Answer: 33,010 g

VIDEO EXAMPLE

Convert 98°F to °C.

SCAN FOR VIDEO WALKTHROUGH
youtu.be/GxLoFH08wSw?si=8MkQXeC19RbVTljY&t=9

Answer: 36.7°C

VIDEO EXAMPLE

Convert 48°C to °F.

SCAN FOR VIDEO WALKTHROUGH
youtu.be/GxLoFH08wSw?si=BC6_pVHDHVa9XTMT&t=113

Answer: 118.4°F

Example 1. Convert the measurement: $8 \text{ m} =$ ____________ cm

Answer: 800 cm

Use the conversion fact $1 \text{ m} = 100 \text{ cm}$.

Since centimeters are smaller than meters, the number gets larger by a factor of 100: $8 \times 100 = 800$.

So $8 \text{ m} = 800 \text{ cm}$.

Example 2. How many cups are in $4\frac{1}{4}$ gallons?

Answer: 68 cups

Use the conversion fact $1 \text{ gal} = 16 \text{ cups}$.

Rewrite $4\frac{1}{4}$ gallons as 4.25 gallons, then multiply: $4.25 \times 16 = 68$.

So $4\frac{1}{4}$ gallons is 68 cups.

Example 3. Apollo Spas services 193 hot tubs. If each hot tub needs 165 mL of muriatic acid, how many liters of acid are needed for all of the hot tubs?

Answer: 31.845 L

Use the conversion fact $1 \text{ L} = 1000 \text{ mL}$.

Set up the dimensional-analysis chain: $193 \text{ tubs} \times 165 \text{ mL per tub} = 31{,}845 \text{ mL}$.

Then convert milliliters to liters: $31{,}845 \div 1000 = 31.845 \text{ L}$.

Example 4. Convert $41°\text{F}$ to Celsius. Round your answer to the nearest tenth.

Answer: $5.0°\text{C}$

Use the temperature formula $C = \frac{5}{9}(F - 32)$.

Substitute $F = 41$: $C = \frac{5}{9}(41 - 32) = \frac{5}{9}(9) = 5$.

Rounded to the nearest tenth, the temperature is $5.0°\text{C}$.

Example 5. Convert the measurement: $339 \text{ cm} =$ ____________ m

Answer: 3.39 m

Use the metric conversion fact $100 \text{ cm} = 1 \text{ m}$.

So convert centimeters to meters by dividing by 100: $339 \div 100 = 3.39$.

That gives $339 \text{ cm} = 3.39 \text{ m}$.

Example 6. Convert the measurement. Round your answer to 3 decimal places as needed: $456 \text{ oz} \approx$ ____________ ton

Answer: 0.014 ton

Use $1 \text{ lb} = 16 \text{ oz}$ and $1 \text{ ton} = 2000 \text{ lb}$, so $1 \text{ ton} = 32{,}000 \text{ oz}$.

Convert ounces to tons: $456 \div 32{,}000 = 0.01425$.

Rounded to three decimal places, $456 \text{ oz} \approx 0.014 \text{ ton}$.

Module 2: Geometry

Geometry examples on angle reasoning, right triangles, perimeter, area, and volume.

VIDEO EXAMPLE

A right triangle has a hypotenuse of 29 cm and one leg of 20 cm. What is the length of the missing leg?

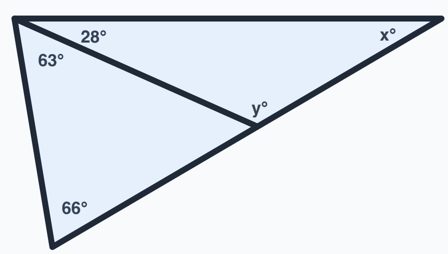
SCAN FOR VIDEO WALKTHROUGH

www.youtube.com/watch?v=goFvYjGYU18&t=1349s

Answer: 21 cm

Example 1. Find the unknown angle measures x and y.

The large triangle shows angles 28 degrees and 63 degrees at the top-left split by an interior segment, 66 degrees at the bottom-left, and unknown angles x and y on the right side.



The figure may not be drawn to scale.

Answer: $x = 23^\circ$, $y = 129^\circ$

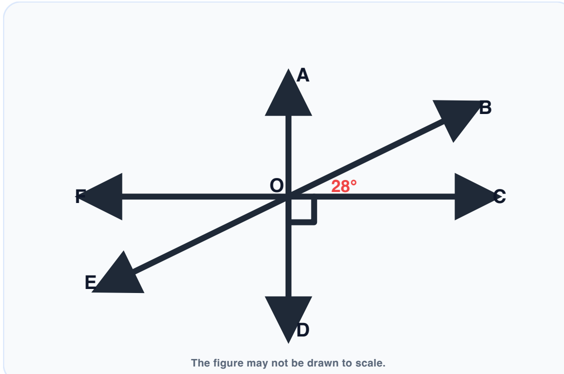
First use the left triangle. Its three angles are 63°, 66°, and the lower angle at the intersection point.

That lower angle is $180 - 63 - 66 = 51^\circ$. Since that angle and y form a straight line, $y = 180 - 51 = 129^\circ$.

Now use the top triangle: $28 + 129 + x = 180$, so $x = 23^\circ$.

Example 2. Use the diagram to identify the angle supplementary to $\angle BOC$, the angle complementary to $\angle BOC$, and the measures of $\angle EOF$, $\angle AOE$, and $\angle BOF$.

A horizontal line, vertical line, and diagonal line all intersect at point O. Arrowheads mark both ends of each line. Angle BOC is 28 degrees and angle COD is a right angle.



Answer: Supplementary: $\angle COE$; **Complementary:** $\angle AOB$; $m\angle EOF = 28°$; $m\angle AOE = 118°$; $m\angle BOF = 152°$

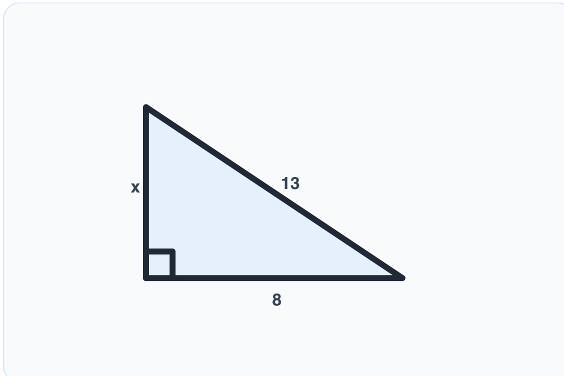
Because $\angle COD$ is a right angle, $\angle AOC$ is also $90°$. Since $\angle BOC = 28°$, the complementary angle is $\angle AOB = 90 - 28 = 62°$.

Supplementary angles add to $180°$, so the angle supplementary to $\angle BOC$ is $\angle COE$. Vertical angles also show that $m\angle EOF = 28°$.

Then $m\angle AOE = 90 + 28 = 118°$ and $m\angle BOF = 180 - 28 = 152°$.

Example 3. Find the length of the leg x. Enter the exact value, not a decimal approximation.

A right triangle shows a vertical leg labeled x, a horizontal leg labeled 8, and a hypotenuse labeled 13.



Answer: $x = \sqrt{105}$

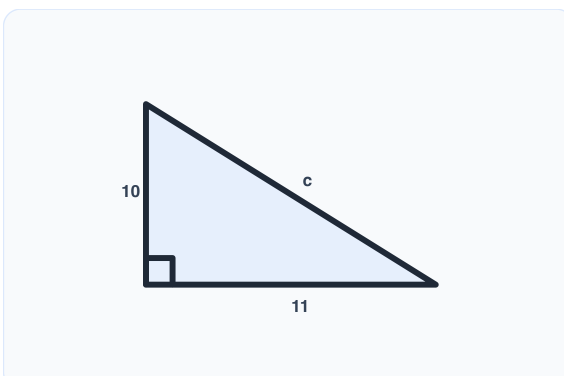
Use the Pythagorean Theorem: $x^2 + 8^2 = 13^2$.

That gives $x^2 + 64 = 169$, so $x^2 = 105$.

Therefore $x = \sqrt{105}$. This radical does not simplify any further, so $\sqrt{105}$ is the exact answer.

Example 4. Find the length of the hypotenuse. Enter the exact value as a square root. Do not round.

A right triangle shows side lengths 10 and 11 on the legs, with the hypotenuse labeled c.



Answer: $c = \sqrt{221}$

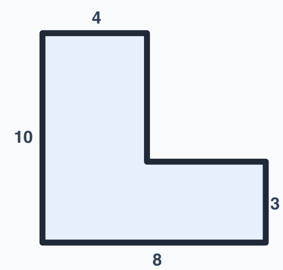
Apply the Pythagorean Theorem: $10^2 + 11^2 = c^2$.

That gives $100 + 121 = c^2$, so $221 = c^2$.

Take the square root to get $c = \sqrt{221}$. Since $221 = 13 \cdot 17$, the radical does not simplify further.

Example 5. Find the perimeter of the figure pictured below.

An L-shaped figure has a top horizontal side of 4, a left vertical side of 10, a bottom side of 8, and a short right side of 3.



The figure may not be drawn to scale.

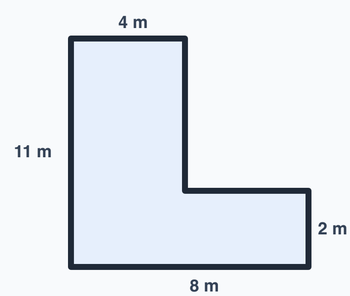
Answer: 36

To get the perimeter, first find the missing inside lengths. The horizontal notch is $8 - 4 = 4$, and the vertical notch is $10 - 3 = 7$.

Now add all six outside sides: $4 + 7 + 4 + 3 + 8 + 10 = 36$. So the perimeter is 36 units.

Example 6. Find the area of the figure pictured below.

An L-shaped figure shows 4 meters across the top, 11 meters on the left side, 8 meters on the bottom, and 2 meters on the short right side.



The figure may not be drawn to scale.

Answer: 52 m^2

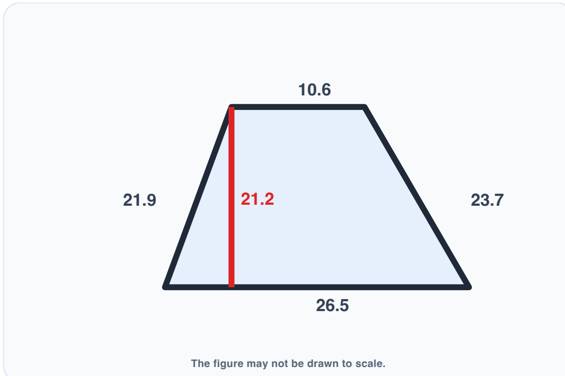
Break the L-shape into two rectangles. The tall rectangle has area $4 \times 11 = 44 \text{ m}^2$.

The short bottom-right rectangle has width $8 - 4 = 4$ meters and height 2 meters, so its area is $4 \times 2 = 8 \text{ m}^2$.

Add them together: $44 + 8 = 52 \text{ m}^2$.

Example 7. Find the perimeter of the trapezoid shown below.

A trapezoid shows a top base of 10.6, bottom base of 26.5, left side of 21.9, right side of 23.7, and a height marked 21.2.



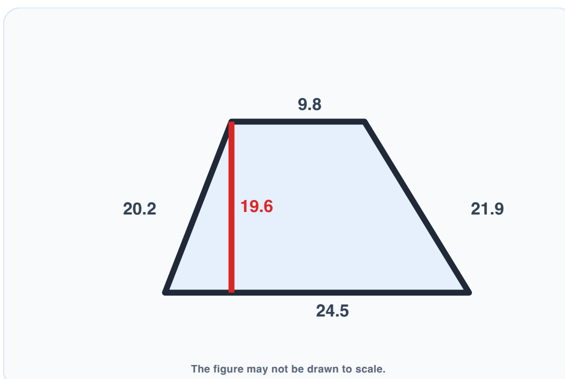
Answer: 82.7

A perimeter problem uses the outside boundary only, so add the four side lengths.

$21.9 + 10.6 + 23.7 + 26.5 = 82.7$. The height is extra information for area, not perimeter.

Example 8. Find the area of the trapezoid shown below.

A trapezoid shows bases of 9.8 and 24.5 with a height marked 19.6 in red.



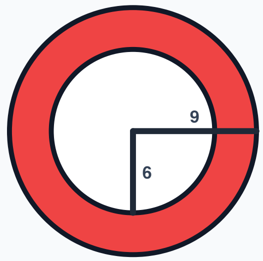
Answer: 336.14

Use the trapezoid area formula: $A = \frac{1}{2}(b_1 + b_2)h$.

Substitute the values: $A = \frac{1}{2}(9.8 + 24.5)(19.6) = \frac{1}{2}(34.3)(19.6) = 336.14$.

Example 9. Find the area of the shaded region. Round your answer to the nearest tenth. Use 3.14 for π.

A ring-shaped shaded region is formed by an outer circle of radius 9 and an inner circle of radius 6.



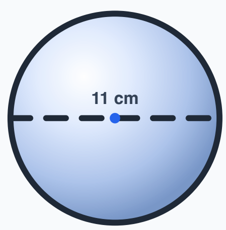
Answer: 141.3 square units

The shaded region is the area of the large circle minus the area of the small circle.

Using 3.14 for π, $A = 3.14(9^2) - 3.14(6^2) = 3.14(81 - 36) = 3.14(45) = 141.3$ square units.

Example 10. A sports ball has a diameter of 11 cm. Find the volume of the ball. Round your answer to 2 decimal places and use 3.14 as an approximation of π.

A shaded sphere is shown as a circle with a dashed horizontal diameter through the center. The diameter is labeled 11 centimeters.



Answer: $696.56\ \text{cm}^3$

For a sphere, use $V = \frac{4}{3}\pi r^3$. The radius is half the diameter, so $r = 5.5$ cm.

Using $\pi \approx 3.14$, $V = \frac{4}{3}(3.14)(5.5)^3 \approx 696.56\ \text{cm}^3$. Round to two decimal places.

Module 3: Problem Solving

Problem-solving examples with multi-step setups and applied word problems.

VIDEO EXAMPLE

The grocery store has bulk almonds on sale. If 6 almond cakes need (1 frac{3}{4}) cups each, how many pounds of almonds do you need?

SCAN FOR VIDEO WALKTHROUGH
youtu.be/o2juWPWlVT0?si=nveQzX3cSz9LAxmi

Nutrition Facts

Servings per Container: 9
Serving Size: 1/4 cup (35 g)

Amount Per Serving	
Calories 180	
	% Daily Value*
Total Fat 15g	19%
Saturated Fat 1g	5%
Trans Fat 0g	
Cholesterol 0mg	0%
Sodium 0mg	0%
Total Carbohydrate 7g	3%
Dietary Fiber 4g	14%
Total Sugars 0g	
Includes 0g Added Sugars	0%
Protein 7g	14%

* The % Daily Value tells you how much a nutrient in a serving of food contributes to a daily diet.

Answer: (3.2 ext{ pounds})

Example 1. Janine is considering buying a water filter and a reusable water bottle rather than buying bottled water. She drinks 6 bottles of water per day, and each bottle is 16.9 oz. She buys 24-packs of 16.9 oz bottles for $3.39. A reusable water bottle costs about $10. A faucet-mounted filter costs about $28 and includes one cartridge; refill filters cost $33 for a 3-pack, and each filter treats up to 100 gallons. A water filter pitcher costs about $22 and includes one cartridge; refill filters cost $20 for a 4-pack, and each filter treats up to 40 gallons. An under-sink filter costs $130 and includes one cartridge; refill filters cost $60 each, and each filter treats up to 500 gallons. Compare the options over 365 days, give your answer to the nearest cent, and pro-rate any partial refill filter use. Will switching from bottled water to a reusable bottle plus filter save Janine money?

Answer: **Faucet-mounted filter; $250.53 saved over one year**

First find how much water Janine drinks in one year: $6(16.9) = 101.4$ ounces per day, so $101.4 \times 365 = 37{,}011$ ounces per year. Since 128 ounces is 1 gallon, she drinks $37{,}011 \div 128 \approx 289.15$ gallons per year.

Bottled water costs $\$3.39 \div 24 = \$0.14125$ per bottle. At 6 bottles per day, the yearly bottled-water cost is $6 \times 365 \times 0.14125 = \$309.34$.

Now compare the filter options, including the reusable bottle. Faucet-mounted: $\$28 + \$10 + (289.15 - 100) \div 100 \times 11 \approx \$58.81$. Pitcher: $\$22 + \$10 + (289.15 - 40) \div 40 \times 5 \approx \$63.14$. Under-sink: $\$130 + \$10 = \$140.00$ because the included 500-gallon filter lasts the whole year.

The faucet-mounted filter is the cheapest option. It saves about $\$309.34 - \$58.81 = \$250.53$ over one year compared with bottled water.

Example 2. Sound travels about 750 miles per hour. If you stand in a canyon and sound a horn, you will hear an echo. How far away is the canyon wall if the echo returns in 2.5 seconds, and what is the distance in terms of n seconds?

Answer: 1375 feet; **in general,** $550n \text{ feet}$

Convert the speed to feet per second: $750 \text{ mi/hr} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ hr}}{3600 \text{ s}} = 1100 \text{ ft/s}$.

The echo time is a round trip, so in 2.5 seconds the sound travels $1100 \times 2.5 = 2750$ feet total. The canyon wall is half that distance away: $2750 \div 2 = 1375$ feet.

If the echo takes n seconds, then the sound travels $1100n$ feet round trip, so the one-way distance to the wall is $\frac{1100n}{2} = 550n$ feet.

Example 3. A long year-end status report for work is 103 pages long, and you need to print 12 copies for a meeting next week. Paper is sold in reams of 500 pages for $3.80 each. How much will the paper cost? Round to the nearest cent.

Answer: $\$9.39$

First find the total number of pages: $103 \times 12 = 1236$ pages.

A ream has 500 pages and costs $\$3.80$, so the cost per page is $\$3.80 \div 500 = \$0.0076$.

Now multiply by the number of pages used: $1236 \times 0.0076 = 9.3936$. Rounded to the nearest cent, the paper cost is $\$9.39$.

Example 4. The grocery store has bulk pecans on sale, and you are planning on making 8 pecan pies for a wedding. Your recipe calls for $1\frac{1}{4}$ cups of pecans per pie, but you only have a scale. Use the nutrition label below to determine how many pounds of pecans you should buy. Give your answer to at least one decimal place.

Nutrition Facts

Serving Size: 1 cup, halves (99 g)
Servings per Container: about 2

Amount Per Serving	
Calories 684	Calories from Fat 596
	% Daily Value*
Total Fat 71g	**110%**
Saturated Fat 6g	**31%**
Trans Fat	
Cholesterol 0mg	**0%**

Answer: 2.2 pounds

Each pie needs $1\frac{1}{4} = 1.25$ cups of pecans, so for 8 pies you need $8 \times 1.25 = 10$ cups.

The nutrition label says 1 cup weighs 99 grams, so 10 cups weighs $10 \times 99 = 990$ grams.

Since 1 pound is about 453.6 grams, $990 \div 453.6 \approx 2.18$ pounds. To at least one decimal place, you should buy 2.2 pounds of pecans.

Example 5. You are planning on making 5 meatloafs for a party. Your recipe calls for $1\frac{1}{4}$ cups of breadcrumbs and makes 1 meatloaf. The canister is cylindrical, 3.5 inches across and 7 inches tall. The net weight of the contents is 15 ounces, and each canister costs $2.19. The nutrition label shows the serving size is $\frac{1}{3}$ cup $(30\ \mathrm{g})$ and there are about 14 servings per container. Give your answer accurate to at least one decimal place. You can only buy whole canisters, but the decimal answer shows how much will be left over. How many canisters of breadcrumbs are needed for 5 meatloafs?

Nutrition Facts

Serving Size: 1/3 cup (30 g)
Servings per Container: about 14

Amount Per Serving	
Calories 110	Calories from Fat 15
	% Daily Value*
Total Fat 1.5g	**2%**

Answer: 1.3 canisters

Each meatloaf needs $1\frac{1}{4} = 1.25$ cups of breadcrumbs, so for 5 meatloafs you need $5 \times 1.25 = 6.25$ cups.

Each canister has about 14 servings of $\frac{1}{3}$ cup, so one canister holds $14 \times \frac{1}{3} = \frac{14}{3} \approx 4.67$ cups.

Now divide the total breadcrumbs needed by the amount in one canister: $6.25 \div 4.67 \approx 1.34$. To at least one decimal place, that is 1.3 canisters, so in practice you would need to buy 2 whole canisters.

Example 6. A 6-inch personal pizza has 630 calories, with 240 of those from fat. A 16-inch pizza is cut into 8 slices. Estimate the number of calories in one slice of the 16-inch pizza.

Answer: 560 calories

Pizza calories are best estimated by area, and the area of a circle depends on the square of the radius. A 6-inch pizza has radius 3, and a 16-inch pizza has radius 8.

So the area scale factor is $\frac{8^2}{3^2} = \frac{64}{9}$. Multiply the calories by that factor to estimate the full 16-inch pizza: $630 \times \frac{64}{9} = 4480$ calories.

The large pizza is cut into 8 slices, so one slice has about $4480 \div 8 = 560$ calories.

Example 7. When ibuprofen is given for fever to children 6 months of age up to 2 years, the usual dose is 5 milligrams (mg) per kilogram (kg) of body weight when the fever is under 102.5 degrees Fahrenheit. What is the usual ibuprofen dose for an 18-month-old weighing 29 pounds? Round to the nearest milligram.

Answer: $66\ \mathrm{mg}$

The dosage is given in milligrams per kilogram, so first convert the child's weight from pounds to kilograms: $29 \div 2.2 \approx 13.18\ \mathrm{kg}$.

Now apply the dosage rule: $13.18 \times 5 \approx 65.9\ \mathrm{mg}$. Rounded to the nearest milligram, the usual dose is $66\ \mathrm{mg}$.

Example 8. You need to buy some chicken for dinner tonight. The store across town has it on sale for $3.19 a pound, while your usual neighborhood store sells it for $3.29 a pound. You will be buying 6 pounds of chicken. Your neighborhood store is 2.1 miles away and takes about 7 minutes. The store across town is 8.5 miles away and takes about 24 minutes. Your car averages about 22 miles per gallon in the city, and gas costs about $3.59 per gallon right now. Is it worth driving across town for the chicken sale? Give monetary values to the nearest cent.

Answer: **Going to the close store is cheaper;** $\$1.49$ **saved**

First compare the chicken prices. The close store costs $6 \times \$3.29 = \$19.74$, and the farther store costs $6 \times \$3.19 = \$19.14$.

Now include gas for the round trip. Close store: $2(2.1) = 4.2$ miles, so gas costs $4.2 \div 22 \times \$3.59 \approx \$0.69$. Farther store: $2(8.5) = 17$ miles, so gas costs $17 \div 22 \times \$3.59 \approx \$2.78$.

Total cost is about $\$19.74 + \$0.69 = \$20.43$ for the close store and $\$19.14 + \$2.78 = \$21.92$ for the farther store. Going to the close store is cheaper, and it saves about $\$21.92 - \$20.43 = \$1.49$.

Example 9. A friend has an 80% average before the final exam for a course. That score includes everything but the final, which counts for 25% of the course grade. What is the best possible course grade, and what final score is needed for a 75% course grade? Give each percentage to one decimal place.

Answer: **Best possible course grade:** 85.0%**; minimum final score for a** 75% **course grade:** 60.0%

The current 80% average applies to the 75% of the course that is already complete, so that part contributes $0.75(80) = 60$ points toward the final course grade.

If your friend earns 100% on the final, the best possible course grade is $60 + 0.25(100) = 85$, so the highest overall grade is 85.0%.

For a 75% course grade, solve $60 + 0.25x = 75$. Then $0.25x = 15$, so $x = 60$. The minimum final-exam score needed is 60.0%.

Example 10. It takes a hose 2 minutes to fill a rectangular aquarium 8 inches long, 9 inches wide, and 11 inches tall. How long will it take to fill a larger aquarium that is 22 inches long, 27 inches wide, and 31 inches tall? Round to the nearest minute.

Answer: 47 minutes

The fill time is proportional to volume. The smaller aquarium has volume $8 \times 9 \times 11 = 792$ cubic inches.

The larger aquarium has volume $22 \times 27 \times 31 = 18{,}414$ cubic inches, so it is $18{,}414 \div 792 = 23.25$ times as large.

Multiply the time by the same factor: $2 \times 23.25 = 46.5$ minutes. Rounded to the nearest minute, it will take 47 minutes.

Example 11. The store is selling lemons at $0.54 each. Each lemon yields about 2 tablespoons of juice. If each pie needs $\frac{1}{2}$ cup of lemon juice, how much will enough lemons cost for two lemon pies?

Answer: $\$4.32$

Each pie needs $\frac{1}{2}$ cup of juice, so two pies need 1 cup total. Since 1 cup is 16 tablespoons, you need 16 tablespoons of lemon juice.

Each lemon gives about 2 tablespoons, so the number of lemons needed is $16 \div 2 = 8$.

At $\$0.54$ per lemon, the total cost is $8 \times \$0.54 = \$4.32$.

Example 12. A company had sales of $330,000 in Seattle in 2012 and $230,000 in Portland in 2012. Compare Seattle and Portland sales using percentages. Round each percentage to the nearest tenth of a percent.

Answer: a. 43.5%; b. 30.3%; c. 69.7%

The difference in sales is $\$330{,}000 - \$230{,}000 = \$100{,}000$.

For part a, compare that difference to Portland's sales: $100{,}000 \div 230{,}000 \approx 0.4348$, so Seattle's sales were 43.5% larger than Portland's.

For part b, compare the same difference to Seattle's sales: $100{,}000 \div 330{,}000 \approx 0.3030$, so Portland's sales were 30.3% smaller than Seattle's.

For part c, compare Portland directly to Seattle: $230{,}000 \div 330{,}000 \approx 0.6970$, so Portland's sales were 69.7% of Seattle's.

Example 13. You read online that a 15 ft by 20 ft brick patio would cost about $2,275 to have professionally installed. Estimate the cost of a 22 ft by 25 ft brick patio. Round to the nearest dollar.

Answer: $\$4{,}171$

Assume the cost is proportional to area. The original patio has area $15 \times 20 = 300 \text{ ft}^2$, and the new patio has area $22 \times 25 = 550 \text{ ft}^2$.

So the new patio is $550 \div 300 = \frac{11}{6}$ times as large as the original patio.

Multiply the original cost by that factor: $\$2{,}275 \times \frac{11}{6} \approx \$4170.83$. Rounded to the nearest dollar, the estimate is $\$4{,}171$.

Module 4: Growth Models

Growth model examples covering linear, exponential, and logistic behavior.

Module 4A Linear Growth

These examples use a constant amount of change per step, so the recursive rules add the same value each time and the explicit formulas are linear.

Example 1. A population of beetles is growing according to a linear growth model. The initial population is $P_0 = 3$, and the population after 10 weeks is $P_{10} = 53$. Find an explicit formula for the beetle population and determine when it reaches 133.

Answer: $P_n = 3 + 5n$; the population reaches 133 after 26 weeks

Because the model is linear, the population changes by the same amount each week. From week 0 to week 10, the population changes by $53 - 3 = 50$, so the weekly increase is $50 \div 10 = 5$.

That gives the explicit formula $P_n = 3 + 5n$.

To find when the population reaches 133, solve $3 + 5n = 133$. Then $5n = 130$, so $n = 26$. The beetle population reaches 133 after 26 weeks.

Example 2. Consider a population that grows according to the recursive rule $P_n = P_{n-1} + 100$, with initial population $P_0 = 70$. Find P_1, P_2, an explicit formula for the population, and P_{100}.

Answer: $P_1 = 170$, $P_2 = 270$, $P_n = 70 + 100n$, $P_{100} = 10070$

Use the recursive rule directly: $P_1 = 70 + 100 = 170$, and $P_2 = 170 + 100 = 270$.

Since the population increases by a constant 100 each step, the explicit formula is $P_n = 70 + 100n$.

Then $P_{100} = 70 + 100(100) = 10070$.

Example 3. The number of cars sold weekly by a new automobile dealership grows according to a linear growth model. The first week the dealership sold six cars $P_0 = 6$. The second week the dealership sold $P_1 = 10$. Write the recursive and explicit formulas for car sales and find the fourth-week sales.

Answer: Recursive rule: $P_n = P_{n-1} + 4$; explicit rule: $P_n = 6 + 4n$; fourth week: 18 cars

The weekly change is $10 - 6 = 4$, so the recursive rule is $P_n = P_{n-1} + 4$.

That same constant increase gives the explicit formula $P_n = 6 + 4n$.

Since P_0 is the first week, the fourth week is P_3. So $P_3 = 6 + 4(3) = 18$ cars.

Example 4. A city currently has 128 streetlights. As part of an urban renewal program, the city council has decided to install 3 additional streetlights at the end of each week for the next 52 weeks. How many streetlights will the city have after 49 weeks?

Answer: 275 streetlights

This is linear growth because the city adds the same number, 3, each week.

After 49 weeks, the total will be $128 + 3(49) = 128 + 147 = 275$. The city will have 275 streetlights.

Module 4B Exponential Growth And Decay

These examples use a constant percent multiplier, so the recursive rules multiply by the same factor and the explicit formulas use powers.

Example 5. A population grows according to an exponential growth model. The initial population is $P_0 = 16$, and the growth rate is $r = 0.2$. Find P_1, P_2, an explicit formula for P_n, and P_{12}. Round non-integer results to one decimal place.

Answer: $P_1 = 19.2$, $P_2 = 23.0$, $P_n = 16(1.2)^n$, $P_{12} \approx 142.7$

A growth rate of 0.2 means the multiplier each step is $1 + 0.2 = 1.2$.

So $P_1 = 16(1.2) = 19.2$, and $P_2 = 19.2(1.2) = 23.04$, which is about 23.0 to one decimal place.

The explicit formula is $P_n = 16(1.2)^n$.

Then $P_{12} = 16(1.2)^{12} \approx 142.7$.

Example 6. Diseases tend to spread according to the exponential growth model. In the early days of AIDS, the growth factor was around 2.0. In 1983, about 1600 people in the U.S. died of AIDS. If that trend had continued unchecked, how many people would have died in 2004?

Answer: $3{,}355{,}443{,}200$ people

A growth factor of 2.0 means the number doubles each year. From 1983 to 2004 is $2004 - 1983 = 21$ years.

So the model gives $1600(2)^{21} = 1600 \times 2{,}097{,}152 = 3{,}355{,}443{,}200$. That is about $3{,}355{,}443{,}200$ people.

Example 7. Starting in the year 2012, the number of speeding tickets issued each year in Middletown is predicted to grow according to an exponential growth model. During 2012, Middletown issued 160 speeding tickets $P_0 = 160$. Every year thereafter, the number of speeding tickets issued is predicted to grow by 5%. Write the recursive and explicit formulas for speeding tickets and predict the number in 2030. Round the predicted number of tickets to the nearest whole number.

Answer: Recursive formula: $P_n = 1.05P_{n-1}$; explicit formula: $P_n = 160(1.05)^n$; in 2030: about 385 tickets

A 5% increase means the multiplier is 1.05, so the recursive rule is $P_n = 1.05P_{n-1}$.

That gives the explicit formula $P_n = 160(1.05)^n$.

The year 2030 is 18 years after 2012, so $P_{18} = 160(1.05)^{18} \approx 385.06$. That is about 385 tickets.

Example 8. The population of Tacoma in 2000 was about 200 thousand and has been growing by about 8% each year. What will the population of Tacoma be in 2014? Round to the nearest person.

Answer: $587{,}439$ people

An 8% annual growth rate means the multiplier is 1.08. From 2000 to 2014 is 14 years.

So the model is $200{,}000(1.08)^{14} \approx 587{,}438.7$. Rounded to the nearest person, Tacoma's population would be about $587{,}439$.

Example 9. Inflation causes things to cost more and money to buy less. Suppose inflation decreases the value of money by 4% each year. In other words, if you have $1 this year, next year it will only buy $0.96 worth of stuff. How much will $100 buy in 20 years? Round to the nearest cent.

Answer: $\$44.20$

A 4% decrease means the yearly multiplier is 0.96.

So after 20 years, the buying power is $100(0.96)^{20} \approx 44.20$. So \($100\) will buy about $\$44.20$ worth of goods.

Module 4C Logistic Growth

These examples start with percent growth but slow down as the population approaches a carrying capacity, so the recursive update depends on how close the population is to the limit.

Example 10. A population of 60 deer are introduced into a wildlife sanctuary. It is estimated that the sanctuary can sustain up to 600 deer. Absent constraints, the population would grow by 60% per year. Estimate the deer population after one year and after two years. Round to one decimal place when needed.

Answer: $p_1 = 92.4$, $p_2 \approx 139.3$

Use the logistic update rule $p_n = p_{n-1} + rp_{n-1}\left(1 - \frac{p_{n-1}}{K}\right)$ with $r = 0.6$, carrying capacity $K = 600$, and starting population $p_0 = 60$.

Then $p_1 = 60 + 0.6(60)\left(1 - \frac{60}{600}\right) = 60 + 36(0.9) = 92.4$.

For the second year, $p_2 = 92.4 + 0.6(92.4)\left(1 - \frac{92.4}{600}\right) \approx 139.296$, so $p_2 \approx 139.3$.

Example 11. Assume there is a certain population of fish in a pond whose growth is described by the logistic equation. It is estimated that the carrying capacity for the pond is 1800 fish. Absent constraints, the population would grow by 170% per year. Estimate the fish population after one breeding season and after two breeding seasons. Round to one decimal place when needed.

Answer: $p_1 = 1280$, $p_2 \approx 1908.6$

Use the same logistic update rule with $r = 1.7$, carrying capacity $K = 1800$, and starting population $p_0 = 600$.

Then $p_1 = 600 + 1.7(600)\left(1 - \frac{600}{1800}\right) = 600 + 1020\left(\frac{2}{3}\right) = 1280$.

Next, $p_2 = 1280 + 1.7(1280)\left(1 - \frac{1280}{1800}\right) \approx 1908.62$, so $p_2 \approx 1908.6$.

Module 5: Finance

Finance examples on compound interest, annuities, savings, withdrawals, and loans.

Module 5A Compound Interest

These examples use lump-sum deposits and future-value or present-value formulas to move money forward or backward in time.

Example 1. How much would you need to deposit in an account now in order to have $4000 in the account in 15 years? Assume the account earns 4% interest compounded monthly.

Answer: $\$2{,}197.44$

Use the compound-interest formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ with $A = 4000$, $r = 0.04$, $n = 12$, and $t = 15$.

Solve for the present deposit: $P = \frac{4000}{\left(1+\frac{0.04}{12}\right)^{180}}$.

That gives $P \approx 2197.438$, so you would need to deposit about $\$2{,}197.44$ now.

Example 2. You deposit $2000 in an account earning 3% interest compounded monthly. How much will you have in the account in 15 years?

Answer: $\$3{,}134.86$

Use the same compound-interest formula with $P = 2000$, $r = 0.03$, $n = 12$, and $t = 15$.

Then $A = 2000\left(1 + \frac{0.03}{12}\right)^{180}$.

Evaluating gives $A \approx 3134.863$, so the account will have about $\$3{,}134.86$.

Module 5B Annuities

These examples use regular monthly deposits or withdrawals, so the formulas combine the monthly rate with a fixed number of payments.

Example 3. You have $300,000 saved for retirement. Your account earns 5% interest. How much will you be able to pull out each month, if you want to be able to take withdrawals for 15 years?

Answer: $\$2{,}372.38$ each month

This is a withdrawal annuity. Use $PV = PMT\left(\frac{1-(1+i)^{-N}}{i}\right)$ with $PV = 300000$, monthly rate $i = \frac{0.05}{12}$, and $N = 15 \cdot 12 = 180$ months.

Solve for the monthly withdrawal: $PMT = 300000\left(\frac{0.05}{12}\right) / \left(1 - \left(1 + \frac{0.05}{12}\right)^{-180}\right)$.

This gives $PMT \approx 2372.381$, so you can withdraw about $\$2{,}372.38$ each month.

Example 4. Suppose you want to have $800,000 for retirement in 25 years. Your account earns 9% interest. a) How much would you need to deposit in the account each month? b) How much interest will you earn?

Answer: a. $\$713.57$ each month; b. about $\$585{,}928.73$

This is a savings annuity. Use $FV = PMT\left(\frac{(1+i)^N-1}{i}\right)$ with $FV = 800000$, monthly rate $i = \frac{0.09}{12}$, and $N = 25 \cdot 12 = 300$.

Solve for the deposit: $PMT = 800000\left(\frac{0.09}{12}\right) / \left(\left(1 + \frac{0.09}{12}\right)^{300} - 1\right) \approx 713.571$.

So the monthly deposit is about $\$713.57$. Using the unrounded value, the total deposited is about $300(713.5709) = \$214{,}071.27$, so the interest earned is about $\$800{,}000 - \$214{,}071.27 = \$585{,}928.73$.

Example 5. You deposit $400 each month into an account earning 7% interest compounded monthly. a) How much will you have in the account in 15 years? b) How much total money will you put into the account? c) How much total interest will you earn?

Answer: a. $\$126{,}784.92$; b. $\$72{,}000$; c. $\$54{,}784.92$

Use the future-value annuity formula $FV = PMT\left(\frac{(1+i)^N-1}{i}\right)$ with $PMT = 400$, monthly rate $i = \frac{0.07}{12}$, and $N = 15 \cdot 12 = 180$.

Then $FV = 400\left(\frac{\left(1+\frac{0.07}{12}\right)^{180}-1}{0.07/12}\right) \approx 126784.919$, so the account value is about $\$126{,}784.92$.

The total amount deposited is $400 \cdot 180 = \$72{,}000$. Subtracting gives total interest of about $\$126{,}784.92 - \$72{,}000 = \$54{,}784.92$.

Module 5C

Loans And Payments

These examples turn loan balances, interest rates, and payment amounts into monthly-payment or loan-size calculations.

Example 6. You want to buy a $230,000 home. You plan to pay 10% as a down payment, and take out a 30 year loan for the rest. a) How much is the loan amount going to be? b) What will your monthly payments be if the interest rate is 5%? c) What will your monthly payments be if the interest rate is 6%?

Answer: a. $\$207{,}000$; b. $\$1{,}111.22$ per month; c. $\$1{,}241.07$ per month

A 10% down payment on $\$230{,}000$ is $0.10(230000) = \$23{,}000$, so the loan amount is $\$230{,}000 - \$23{,}000 = \$207{,}000$.

For a 30-year mortgage, $N = 30 \cdot 12 = 360$ monthly payments. Use $PMT = PV\, i / \left(1 - (1+i)^{-N}\right)$.

At 5%, $i = \frac{0.05}{12}$ gives $PMT \approx \$1{,}111.22$. At 6%, $i = \frac{0.06}{12}$ gives $PMT \approx \$1{,}241.07$.

Example 7. You have $4,000 on a credit card that charges a 23% interest rate. If you want to pay off the credit card in 5 years, how much will you need to pay each month (assuming you do not charge anything new to the card)?

Answer: $\$112.76$ each month

Treat the payoff plan like a loan payment problem with present value $PV = 4000$, monthly rate $i = \frac{0.23}{12}$, and $N = 5 \cdot 12 = 60$ payments.

Use $PMT = PV\, i / \left(1 - (1+i)^{-N}\right)$.

Substituting the values gives $PMT \approx 112.7619$, so the needed payment is about $\$112.76$ each month.

Example 8. You can afford a $300 per month car payment. You have found a 3 year loan at 6% interest. How big of a loan can you afford?

Answer: $\$9{,}861.30$

This time the monthly payment is known, and you want the loan amount. Use the present-value annuity formula $PV = PMT\left(\frac{1-(1+i)^{-N}}{i}\right)$.

Here $PMT = 300$, $i = \frac{0.06}{12}$, and $N = 3 \cdot 12 = 36$.

So $PV = 300\left(\frac{1-\left(1+\frac{0.06}{12}\right)^{-36}}{0.06/12}\right) \approx 9861.305$. You can afford a loan of about $\$9{,}861.30$.

Module 6: Statistics - Collecting Data

Statistics examples on populations and samples, sampling methods, bias, experiments, and conclusions.

Example 1. A school district wants to estimate the percent of district households that support a later start time for high school. The district randomly surveys 350 households and finds that 61% support the change. Identify the population, sample, parameter, and statistic.

Answer: Population: all district households; sample: the 350 surveyed households; parameter: the true percent of all district households that support the change; statistic: 61%

The population is the full group the study wants information about, and the sample is the part that was actually surveyed.

A parameter describes the population, while a statistic describes the sample.

So here the population is all district households, the sample is the 350 surveyed households, the parameter is the true percent of all district households that support the later start time, and the statistic is the sample result of 61%.

Example 2. Identify the sampling method used in each case. a) A principal writes every student name on identical slips, mixes them, and draws 40 names. b) A city separates residents into north, central, and south neighborhoods, then randomly selects 50 from each neighborhood. c) A researcher randomly chooses 8 homerooms and surveys every student in those homerooms. d) A news site posts a poll and asks readers to click the answer they prefer.

Answer: a. simple random; b. stratified; c. cluster; d. voluntary response

A simple random sample gives every individual an equal chance of selection. A stratified sample splits the population into groups first and samples from each group. A cluster sample randomly selects whole groups and surveys everyone in those groups. A voluntary-response sample relies on people choosing to participate.

That makes the answers: a) simple random, b) stratified, c) cluster, d) voluntary response.

Example 3. For each study, decide whether the conclusion about the larger population is justified. If it is not, state the main problem. a) A restaurant asks diners who used a coupon to rate the restaurant, then concludes all customers are highly satisfied. b) A researcher interested in Springfield shopping habits surveys a randomly selected group of 200 Walmart shoppers. 76% say price matters more than where an item was produced. The researcher concludes that about three quarters of people in Springfield care more about cost than where an item is made. c) A school sends an online survey about homework policy and counts only students who choose to reply.

Answer: a. not justified; coupon users are not representative of all customers; b. not justified; Walmart shoppers are not representative of all Springfield residents; c. not justified; voluntary-response bias

A study can only support a conclusion about the larger population if the sample is reasonably representative of that population.

In part a, coupon users are not necessarily representative of all customers. In part b, Walmart shoppers are not necessarily representative of all Springfield residents. In part c, students who choose to reply to an online survey create voluntary-response bias.

So none of the three conclusions is justified as stated.

Example 4. For each situation, say whether it is an observational study or an experiment, and say whether a cause-and-effect conclusion is appropriate. a) Researchers record how many hours students sleep and compare that with GPA. b) A doctor randomly assigns patients to receive a new allergy medicine or a placebo. c) A school counselor surveys athletes and non-athletes about stress levels.

Answer: a. observational study; no cause-and-effect conclusion; b. experiment; a cause-and-effect conclusion may be appropriate; c. observational study; no cause-and-effect conclusion

An observational study records what is already happening, while an experiment imposes a treatment.

Cause-and-effect conclusions require a well-designed experiment, not just observed associations.

So a) observational, no cause-and-effect conclusion; b) experiment, a cause-and-effect conclusion may be justified if the design is sound; c) observational, no cause-and-effect conclusion.

Example 5. A company wants to test whether an energy drink improves reaction time. It randomly assigns 80 volunteers to two groups. One group receives the new drink. The other receives a similar-looking drink with no active ingredient. Neither the volunteers nor the person timing reactions knows which drink each volunteer received. Identify the treatment group, control group, placebo, and whether the study is blind or double-blind. Then explain why random assignment matters.

Answer: Treatment group: the group receiving the new drink; control group: the group receiving the similar-looking drink with no active ingredient; placebo: the similar-looking drink with no active ingredient; design: double-blind; random assignment helps create comparable groups and reduce bias

The treatment group is the group that receives the actual condition being tested, and the control group provides the comparison. A placebo looks like the treatment but has no active ingredient.

Since neither the volunteers nor the timer knows who got which drink, the study is double-blind.

Random assignment matters because it helps create comparable groups and reduces bias from pre-existing differences.

Example 6. Each study claims one variable causes another. For each one, decide whether the claim is justified and identify a likely confounding variable. a) A study finds that students who carry water bottles tend to have higher test scores, so the researcher concludes that carrying a water bottle improves academic performance. b) People who sleep fewer than 6 hours report drinking more coffee, so a news article concludes that coffee causes people to lose sleep. c) A city installs new streetlights in one neighborhood, and reported nighttime crime drops there the next month. Officials immediately conclude that the lights caused the drop.

Answer: a. not justified; possible confounder: study habits or organization; b. not justified; possible confounder: stress, workload, or reverse causation; c. not justified; possible confounder: policing changes, weather, or seasonal trends

A confounding variable is another factor that may help explain the observed result, so the claimed cause is not established.

In part a, stronger study habits or general organization could explain both carrying a water bottle and higher scores. In part b, stress, workload, or reverse causation could explain the association between coffee and sleep. In part c, other changes such as policing, weather, or seasonal trends could affect crime.

So none of the three cause-and-effect claims is justified from the information given.

Module 7: Statistics: Describing Data

Statistics examples on graphs, five-number summaries, box plots, and measures of center and spread.

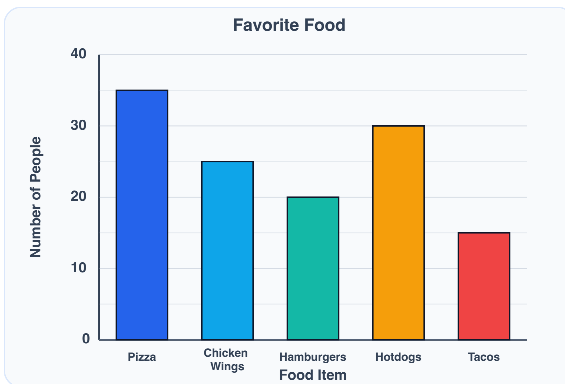
VIDEO EXAMPLE

At a local dive bar, customers were asked what their favorite item was on the menu. What percentage of the people chose chicken wings or tacos?

SCAN FOR VIDEO WALKTHROUGH

www.youtube.com/shorts/qahdnlyq7Os

The bar graph compares how many people chose pizza, chicken wings, hamburgers, hotdogs, and tacos. Pizza has the tallest bar, and tacos has the shortest bar.



Answer: 32%

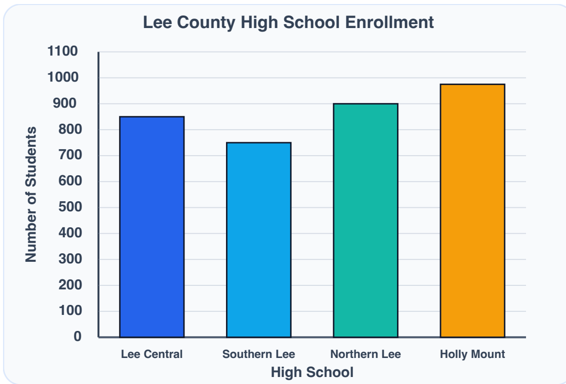
VIDEO EXAMPLE

There are four high schools in Lee County. The enrollment numbers are shown in the graph below. What is the approximate percentage of high school students in Lee County that attends Lee Central High School? Round to the nearest tenth of a percent.

SCAN FOR VIDEO WALKTHROUGH

youtu.be/0LPYxG1zpz4?si=gU8xq7vt36jzBeED&t=17

The bar graph compares student enrollment at Lee Central, Southern Lee, Northern Lee, and Holly Mount on a scale from 0 to 1100 students. Holly Mount has the largest enrollment.



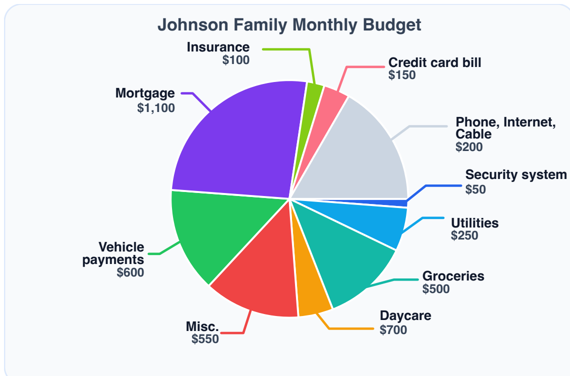
Answer: 24.5%

VIDEO EXAMPLE

The Johnson family's $4,200 monthly budget is shown in the pie chart below. What percent of their monthly budget is spent on groceries? Round to the nearest percent.

SCAN FOR VIDEO WALKTHROUGH
youtu.be/nFbenBviymk?si=wRU4Vj5Dxganmtzi&t=16

The circle graph shows the Johnson family monthly budget split into ten categories. Mortgage is the largest expense at 1,100 dollars, and the security system is the smallest at 50 dollars.



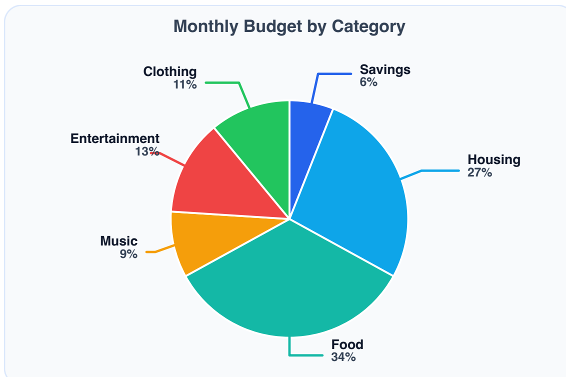
Answer: 12%

VIDEO EXAMPLE

A family brings home $12,200 each month. Use the circle graph to determine how much money is budgeted for each category.

SCAN FOR VIDEO WALKTHROUGH
youtu.be/fipEf9Xzhn4?si=K1mN7s5rGRNQX_AF&t=11

The circle graph shows a monthly budget divided into savings 6 percent, housing 27 percent, food 34 percent, music 9 percent, entertainment 13 percent, and clothing 11 percent. Food is the largest category.



Answer: **Savings: $732; Housing: $3,294; Food: $4,148; Music: $1,098; Entertainment: $1,586; Clothing: $1,342.**

Example 1. The ages of 30 lottery winners are shown below. Complete the frequency distribution for the age classes 20-29, 30-39, 40-49, 50-59, 60-69, 70-79, and 80-89.

Age of 30 lottery winners

21	26	30	33	36	36
37	44	48	51	51	53
54	54	56	59	60	64
65	65	65	69	71	73
75	76	76	76	77	88

AGE	FREQUENCY
20-29	
30-39	
40-49	
50-59	
60-69	
70-79	
80-89	

Answer: 20-29: 2; 30-39: 5; 40-49: 2; 50-59: 7; 60-69: 6; 70-79: 7; 80-89: 1

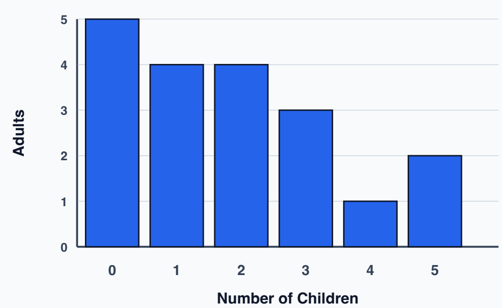
Count how many ages fall in each class.

There are 2 values in the 20s, 5 in the 30s, 2 in the 40s, 7 in the 50s, 6 in the 60s, 7 in the 70s, and 1 in the 80s.

Those frequencies add to 30, so the distribution is complete.

Example 2. A bar graph shows the number of adults who reported each number of children. How many adults were questioned, and what percentage had 0 children? Give the percentage to the nearest tenth of a percent.

The bar graph shows number of children on the x-axis and adults on the y-axis. The bars for 0 through 5 children have heights 5, 4, 4, 3, 1, and 2.



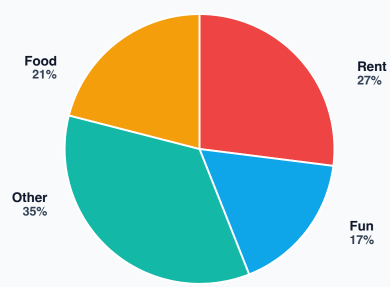
Answer: 19 adults; about 26.3% had 0 children

Add the bar heights: 5 + 4 + 4 + 3 + 1 + 2 = 19 adults.

The bar for 0 children has height 5, so the percentage is $\frac{5}{19} \times 100 \approx 26.3\%$.

Example 3. Luciana categorized her spending for the month into Rent, Food, Fun, and Other as shown in the circle graph. If she spent $2800 this month, how much did she spend on Fun?

The circle graph shows Luciana's monthly spending by category: Rent 27%, Fun 17%, Other 35%, and Food 21%.



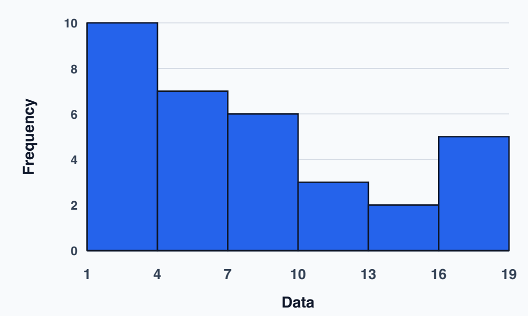
Answer: $476

Fun is 17% of the total spending.

Compute 17% of $2800: $0.17 \times 2800 = 476$. So Luciana spent $476 on Fun.

Example 4. Based on the histogram, what is the class width and what is the sample size?

The histogram shows class boundaries 1, 4, 7, 10, 13, 16, and 19. The six bars have frequencies 10, 7, 6, 3, 2, and 5.



Answer: **Class width: 3; sample size: 33**

The class boundaries increase by 3 each time, so the class width is 3.

Add the frequencies to get the sample size: $10 + 7 + 6 + 3 + 2 + 5 = 33$.

Example 5. Two classes were given identical quizzes. Class A had a mean score of 8.4 and a standard deviation of 0.9. Class B had a mean score of 7.5 and a standard deviation of 0.3. Which class scored better on average, and which class had more consistent scores?

Class A

Mean	8.4
Standard deviation	0.9

Class B

Mean	7.5
Standard deviation	0.3

Answer: **Class A scored better on average; Class B had more consistent scores**

The mean gives the average score, so compare 8.4 and 7.5.

Class A scored better on average because 8.4 is larger. The smaller standard deviation, 0.3, means Class B's scores were more tightly clustered, so Class B was more consistent.

Example 6. Consider the ordered data set 4, 6, 7, 9, 11, 13, 15, 16, 19. Find the mean, median, five-number summary, draw a box plot, and find the sample standard deviation. Round the mean and standard deviation to the nearest tenth.

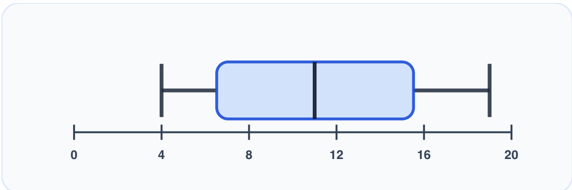
Answer: Mean: 11.1; median: 11; five-number summary: (4, 6.5, 11, 15.5, 19); sample standard deviation: about 5.0

The mean is $\frac{4+6+7+9+11+13+15+16+19}{9} = \frac{100}{9} \approx 11.1.$

With 9 values, the median is the middle value, so the median is 11. The lower half is 4, 6, 7, 9, so $Q_1 = 6.5$. The upper half is 13, 15, 16, 19, so $Q_3 = 15.5$.

That gives the five-number summary $(4, 6.5, 11, 15.5, 19)$. Using the sample standard deviation formula, the standard deviation is about 5.0.

The box plot shows minimum 4, first quartile 6.5, median 11, third quartile 15.5, and maximum 19.



Example 7. Consider the ordered data set 3, 5, 6, 8, 9, 11, 12, 14, 15, 17. Find the mean, median, five-number summary, draw a box plot, and find the sample standard deviation. Round the standard deviation to the nearest tenth.

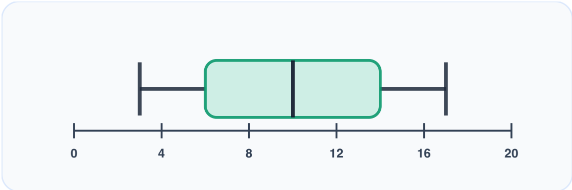
Answer: Mean: 10; median: 10; five-number summary: (3, 6, 10, 14, 17); sample standard deviation: about 4.6

The mean is $\frac{3+5+6+8+9+11+12+14+15+17}{10} = \frac{100}{10} = 10.$

With 10 values, the median is the average of the two middle values: $\frac{9+11}{2} = 10$. The lower half is 3, 5, 6, 8, 9, so $Q_1 = 6$. The upper half is 11, 12, 14, 15, 17, so $Q_3 = 14$.

That gives the five-number summary $(3, 6, 10, 14, 17)$. Using the sample standard deviation formula, the standard deviation is about 4.6.

The box plot shows minimum 3, first quartile 6, median 10, third quartile 14, and maximum 17.



Module 8: Probability

Probability examples on basic probability, unions and intersections, conditional probability, and expected value.

VIDEO EXAMPLE

If you roll a fair six-sided die once, what is the probability of rolling a 5? Round to the nearest percent.

SCAN FOR VIDEO WALKTHROUGH
youtu.be/5C9C-rbTQ1g?si=o84snjlvBtv19bCe&t=252

Answer: 17%

VIDEO EXAMPLE

Jackie has 5 quarters, 10 nickels, 9 pennies, and 1 dime in her purse. If she draws one coin at random, what is the probability that it is a nickel?

SCAN FOR VIDEO WALKTHROUGH
youtu.be/5C9C-rbTQ1g?si=wVOsTPjWWil2vSb8&t=428

Answer: $\frac{2}{5} = 0.4 = 40\%$

Example 1. A group of people were asked if they had run a red light in the last year. 361 responded yes, and 170 responded no. Find the probability that if a person is chosen at random, they have run a red light in the last year. Give your answer as a fraction or decimal accurate to at least 3 decimal places.

Answer: $\frac{361}{531} \approx 0.680$

The total number of responses is $361 + 170 = 531$.

The probability of choosing a person who answered yes is $\frac{361}{531}$, which is about 0.680 to three decimal places.

Example 2. A jar contains 12 red marbles numbered 1 to 12 and 8 blue marbles numbered 1 to 8. A marble is drawn at random from the jar. Find the probability of each event.

(a) The marble is red

(b) The marble is odd-numbered

(c) The marble is red or odd-numbered

(d) The marble is blue and even-numbered

Answer: (a) $\frac{3}{5}$; (b) $\frac{1}{2}$; (c) $\frac{4}{5}$; (d) $\frac{1}{5}$

There are 20 marbles total.

For part (a), 12 of the 20 marbles are red, so $P(\text{red}) = \frac{12}{20} = \frac{3}{5}$.

For part (b), the odd-numbered marbles are 1, 3, 5, and 7, 9, 11 among the red marbles and 1, 3, 5, 7 among the blue marbles, so $P(\text{odd}) = \frac{10}{20} = \frac{1}{2}$.

For part (c), there are 12 red marbles, 10 odd-numbered marbles, and 6 marbles counted in both groups, so $P(\text{red or odd}) = \frac{12+10-6}{20} = \frac{16}{20} = \frac{4}{5}$.

For part (d), the blue even-numbered marbles are 2, 4, 6, and 8, so $P(\text{blue and even}) = \frac{4}{20} = \frac{1}{5}$.

Example 3. Suppose a jar contains 20 red marbles and 34 blue marbles. If you reach in the jar and pull out 2 marbles at random at the same time, find the probability that both are red. Fractions or decimals are acceptable. If you enter a decimal, round to the nearest thousandth.

Answer: $\frac{190}{1431} \approx 0.133$

Because two marbles are pulled at the same time, this is a without-replacement probability.

The probability the first marble is red is $\frac{20}{54}$, and then the probability the second marble is red is $\frac{19}{53}$. Multiply: $\frac{20}{54} \cdot \frac{19}{53} = \frac{190}{1431} \approx 0.133.$

Example 4. The table summarizes results from 988 pedestrian deaths that were caused by automobile accidents. If one of the pedestrian deaths is randomly selected, find the probability that the pedestrian was intoxicated or the driver was not intoxicated. Report the answer as a percent rounded to one decimal place. You need not enter the % symbol.

Driver Intoxicated?	Pedestrian Intoxicated?	
	Yes	No
Yes	63	85
No	235	605

Answer: 91.4

Let A be the event that the pedestrian was intoxicated and B be the event that the driver was not intoxicated.

Using the addition rule, $P(A \text{ or } B) = \frac{298+840-235}{988} = \frac{903}{988} \approx 0.914$. As a percent, that is about 91.4%.

Example 5. A test was given to a group of students. The grades and gender are summarized below. If one student is chosen at random from those who took the test, find the probability that the student got a C given they are male.

	A	B	C	Total
Male	17	20	16	53
Female	14	18	7	39
Total	31	38	23	92

Answer: $\frac{16}{53} \approx 0.302$

The word given means you restrict the sample space first.

Among the male students, 16 earned a C out of 53 male students total. So $P(C \mid \text{male}) = \frac{16}{53} \approx 0.302.$

Example 6. A bag contains 1 gold marble, 10 silver marbles, and 30 black marbles. Someone offers to play this game: You randomly select one marble from the bag. If it is gold, you win $4. If it is silver, you win $3. If it is black, you lose $1. What is your expected value if you play this game? Round your answer to two decimal places.

Answer: **Expected value:** $\$0.10$

Use expected value: multiply each outcome by its probability and add.

Here that gives $4\left(\frac{1}{41}\right)+3\left(\frac{10}{41}\right)+(-1)\left(\frac{30}{41}\right)=\frac{4}{41}\approx 0.10$. The expected value is a gain of about $\$0.10$ per play.

Module 9: The Normal Distribution

Normal distribution examples on z-scores, probabilities, percentiles, and empirical-rule graphs.

Module 9A Empirical Rule And Z-Scores

Start with the 68-95-99.7 rule and the z-score formula before moving into calculator-based normal probability work.

IN PLAIN TERMS

Normal distribution questions usually ask you to either find an area from one or two cutoffs or find a cutoff from a given area or percentile.

KEY PARTS

- Normal problems usually give a mean μ and standard deviation σ.
- Use a left-tail setup for phrases like less than, below, or at most.
- Use a right-tail setup for phrases like greater than, above, top percent, or heaviest percent.
- Use a between setup when the question gives two cutoffs and asks for the middle area.
- Use a cutoff or percentile setup when the area is given but the missing value is x or z.
- For empirical-rule graphs, remember that about 68% of the data are within 1 standard deviation, 95% are within 2, and 99.7% are within 3.

RULES AND FORMULAS

- Standardize raw values with $z = \frac{x-\mu}{\sigma}$.
- TI-84 probability command: **normalcdf(lower, upper, mean, sd)**. When using this command, use 1E99 for the upper when calculating a right-tail probability and use -1E99 for the lower when calculating a left-tail probability.
- TI-84 cutoff command: **invNorm(area, mean, sd)**. Some models of the TI-84 allow you to choose the tail direction directly.
- If the problem wants a probability, use **normalcdf**. If the problem wants a cutoff value from a percent or area, use **invNorm**.
- If you do not have a TI-84, you can use the Normal Distribution calculator to find probabilities and the Inverse Normal Distribution calculator to find cutoff values.

LOOK FOR: whether the problem is asking for an area or a cutoff, and whether the region is left tail, right tail, between two values, or outside two values.

Question 1. The physical plant at the main campus of a large state university receives daily requests to replace florecent lightbulbs. The distribution of the number of daily requests is bell-shaped and has a mean of 38 and a standard deviation of 4. Using the 68-95-99.7 rule, what is the approximate percentage of lightbulb replacement requests numbering between 38 and 46?

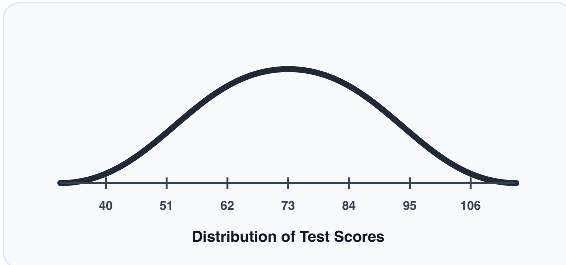
Answer: 47.5%

The value 46 is two standard deviations above the mean because (38 + 2(4) = 46).

From the mean to (+2sigma), the empirical rule gives (34% + 13.5% = 47.5%).

Question 2. The graph illustrates the distribution of test scores taken by College Algebra students. The maximum possible score on the test was 120, while the mean score was 73 and the standard deviation was 11. (34,13.5,2.35,0.15)

A bell-shaped curve is centered at 73 with equally spaced tick marks every 11 points from 40 through 106.



(a) What is the approximate percentage of students who scored between 51 and 95 on the test?

(b) What is the approximate percentage of students who scored between 62 and 84 on the test?

(c) What is the approximate percentage of students who scored higher than 95 on the test?

(d) What is the approximate percentage of students who scored lower than 40 on the test?

Answer: **(a) 95%; (b) 68%; (c) 2.5%; (d) 0.15%**

Use the empirical-rule segment values noted in the homework: 34, 13.5, 2.35, and 0.15.

That gives (a) 95%, (b) 68%, (c) 2.5%, and (d) 0.15%.

Question 3. The graph illustrates a normal distribution for the prices paid for a particular model of HD television. The mean price paid is $1200 and the standard deviation is $95.

A bell-shaped curve is centered at 1200 with equally spaced tick marks every 95 dollars from 915 through 1485.

(a) What is the approximate percentage of buyers who paid between $1200 and $1390?

(b) What is the approximate percentage of buyers who paid between $1105 and $1200?

(c) What is the approximate percentage of buyers who paid between $915 and $1200?

(d) What is the approximate percentage of buyers who paid between $1105 and $1295?

(e) What is the approximate percentage of buyers who paid less than $915?

(f) What is the approximate percentage of buyers who paid less than $1010?

Answer: (a) 47.5%; (b) 34%; (c) 49.85%; (d) 68%; (e) 0.15%; (f) 2.5%

Each tick mark is one standard deviation, so the homework is using the 68-95-99.7 rule on the labeled price graph.

That gives (a) 47.5%, (b) 34%, (c) 49.85%, (d) 68%, (e) 0.15%, and (f) 2.5%.

Question 4. A normal distribution has a mean of 102 and a standard deviation of 6. Find the z-score for a data value of 126. Round to two decimal places.

Answer: 4.00

Use the z-score formula: (z = rac{x - mu}{sigma} = rac{126 - 102}{6} = 4).

Module 9B

Normal Probabilities

Use left-tail, right-tail, and between setups to calculate probabilities from raw values and standard-normal z-values.

Question 5. The lengths of pregnancies in a small rural village are normally distributed with a mean of 261 days and a standard deviation of 16 days. What percentage of pregnancies last beyond 213 days? Enter your answer as a percent accurate to 1 decimal place.

Answer: 99.9%

This is a probability from a cutoff, so use the probability tool from the **Rules and Formulas** section above.

This one is a **right-tail / Above** probability starting at **213**.

That area is about (0.99865), which is (99.9%).

Question 6. Suppose your manager indicates that for a normally distributed data set you are analyzing, your company wants data points between (z = -1.2) and (z = 1.2) standard deviations of the mean (or within 1.2 standard deviations of the mean). What percent of the data points will fall in that range?

Answer: 76.99%

This is a probability between two cutoffs, so use the probability tool from the **Rules and Formulas** section above.

Set it up as a **between** probability with cutoffs (-1.2) and (1.2).

That difference is about (0.7698607), so about (76.99%) of the data fall in the range.

Question 7. For the standard normal distribution, find: (P(-1.66 < z < 0.43)).

Answer: 0.6179

This is another probability between two cutoffs, so use the probability tool from the **Rules and Formulas** section above.

Set it up as a **between** probability with cutoffs (-1.66) and (0.43).

That gives (P(-1.66 < Z < 0.43) approx 0.6179).

Question 8. An electronic product takes an average of 3 hours to move through an assembly line. If the standard deviation of 0.4 hours, what is the probability that an item will take between 3.6 and 3.7 hours to move through the assembly line? Do not round until you get your final answer, and then round to 3 decimal places.

Answer: 0.027

This is a probability between two raw values, so use the probability tool from the **Rules and Formulas** section above.

Set it up as a **between** probability with mean **3**, standard deviation **0.4**, and cutoffs **3.6** and **3.7**.

That probability is about 0.026748, which rounds to 0.027.

Question 9. The systolic blood pressure of adults in the USA is nearly normally distributed with a mean of 119 and standard deviation of 24. Someone qualifies as having Stage 2 high blood pressure if their systolic blood pressure is 160 or higher.

(a) Around what percentage of adults in the USA have stage 2 high blood pressure? Give your answer rounded to two decimal places.

(b) If you sampled 2000 people, how many would you expect to have BP > 160? Give your answer to the nearest person.

(c) Stage 1 high BP is specified as systolic BP between 140 and 160. What percentage of adults in the US qualify for stage 1?

(d) Your doctor tells you you are in the 30th percentile for blood pressure among US adults. What is your systolic BP? Round to 2 decimal places.

Answer: **(a) 4.38%; (b) 88 people; (c) 14.70%; (d) 106.41**

Parts (a) and (c) are probability questions, so use the probability tool from the **Rules and Formulas** section above. Part (a) is a **right-tail / Above** setup at **160**, and part (c) is a **between** setup from **140** to **160**.

Part (a) gives an upper-tail percentage of about 4.38%, and part (b) uses expected count: $2000 \times 0.043787 \approx 87.6$, so about 88 people.

For part (c), subtracting the cumulative values gives $P(140 < X < 160) \approx 14.70\%$.

Part (d) is a percentile cutoff, so use the cutoff tool from the **Rules and Formulas** section above. This one uses the **left tail / Below** area of **0.30**, which gives $x \approx 106.41$.

Module 9C Percentiles And Cutoff Values

Work backward from a percent or area to find the matching z-score or raw value on a normal distribution.

Question 10. For a standard normal distribution, find: (P(z > c) = 0.6041). Find (c).

Answer: (c approx -0.264)

This is a cutoff from a **right-tail** area.

Use the cutoff tool from the **Rules and Formulas** section above. Since the given area is on the right, use a **right-tail / Above** area of **0.6041** if that option is available, or convert it to the equivalent left-tail area of (0.3959).

Either way, the cutoff is about (-0.264).

Question 11. A particular fruit's weights are normally distributed, with a mean of 646 grams and a standard deviation of 11 grams. The heaviest 16% of fruits weigh more than how many grams? Give your answer to the nearest gram.

Answer: 657 grams

This is a cutoff from the **right tail** because the heaviest 16% are above the answer.

Use the cutoff tool from the **Rules and Formulas** section above. Set it up with a **right-tail / Above** area of **0.16**, or use the equivalent **left-tail / Below** area of **0.84**.

That gives a cutoff of about (656.94), so the answer is (657) grams.

Question 12. Engineers must consider the diameters of heads when designing helmets. The company researchers have determined that the population of potential clientele have head diameters that are normally distributed with a mean of 6.4-in and a standard deviation of 0.8-in. Due to financial constraints, the helmets will be designed to fit all men except those with head diameters that are in the smallest 0.7% or largest 0.7%. What is the minimum head diameter that will fit the clientele? What is the maximum head diameter that will fit the clientele? Enter your answer as a number accurate to 1 decimal place.

Answer: Minimum: 4.4 in; maximum: 8.4 in

This is a cutoff problem, so use the cutoff tool from the **Rules and Formulas** section above twice.

For the minimum, use the **left tail / Below** area of **0.007**. For the maximum, use the matching **right tail / Above** area of **0.007**, or the equivalent **left-tail / Below** area of **0.993**.

That gives about (4.43) and (8.37), which round to (4.4) in and (8.4) in.